admcycles

A Sage-package for computations in the cohomology ring of the moduli space of stable curves

Johannes Schmitt

July 2019

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The moduli space of stable curves

Definition (Deligne-Mumford 1969)

Let $g, n \geq 0$ be integers (with $2g - 2 + n > 0$).

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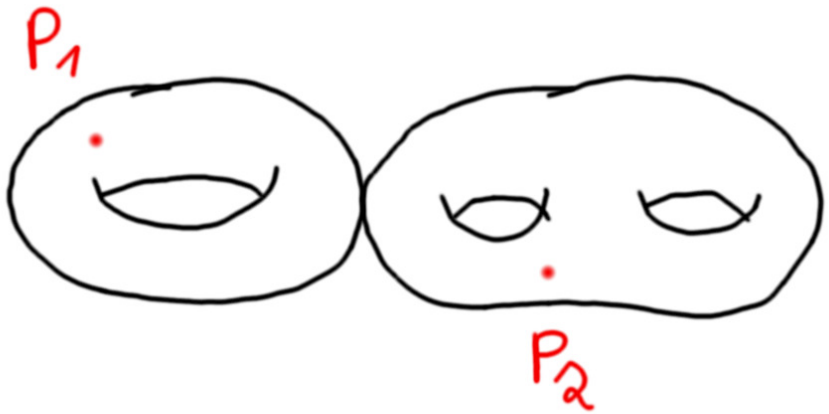

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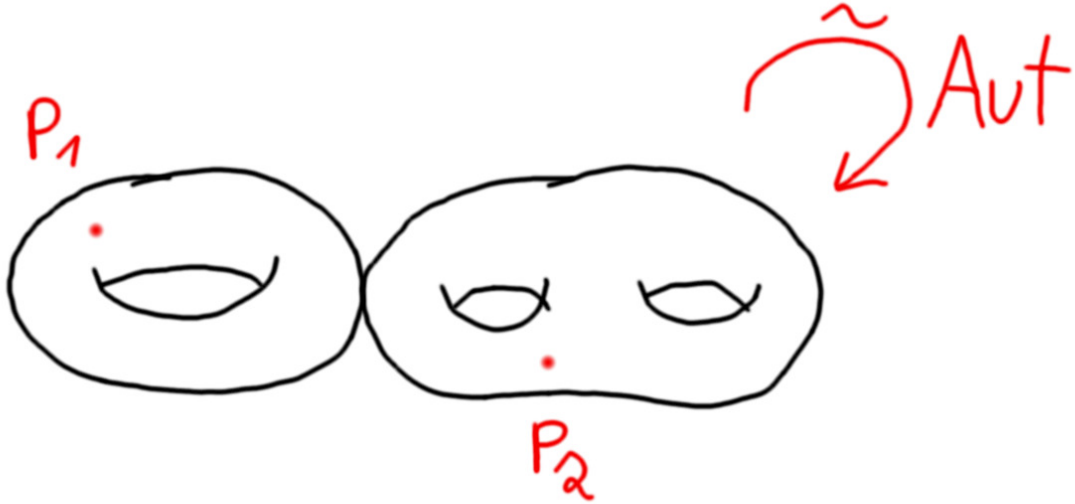

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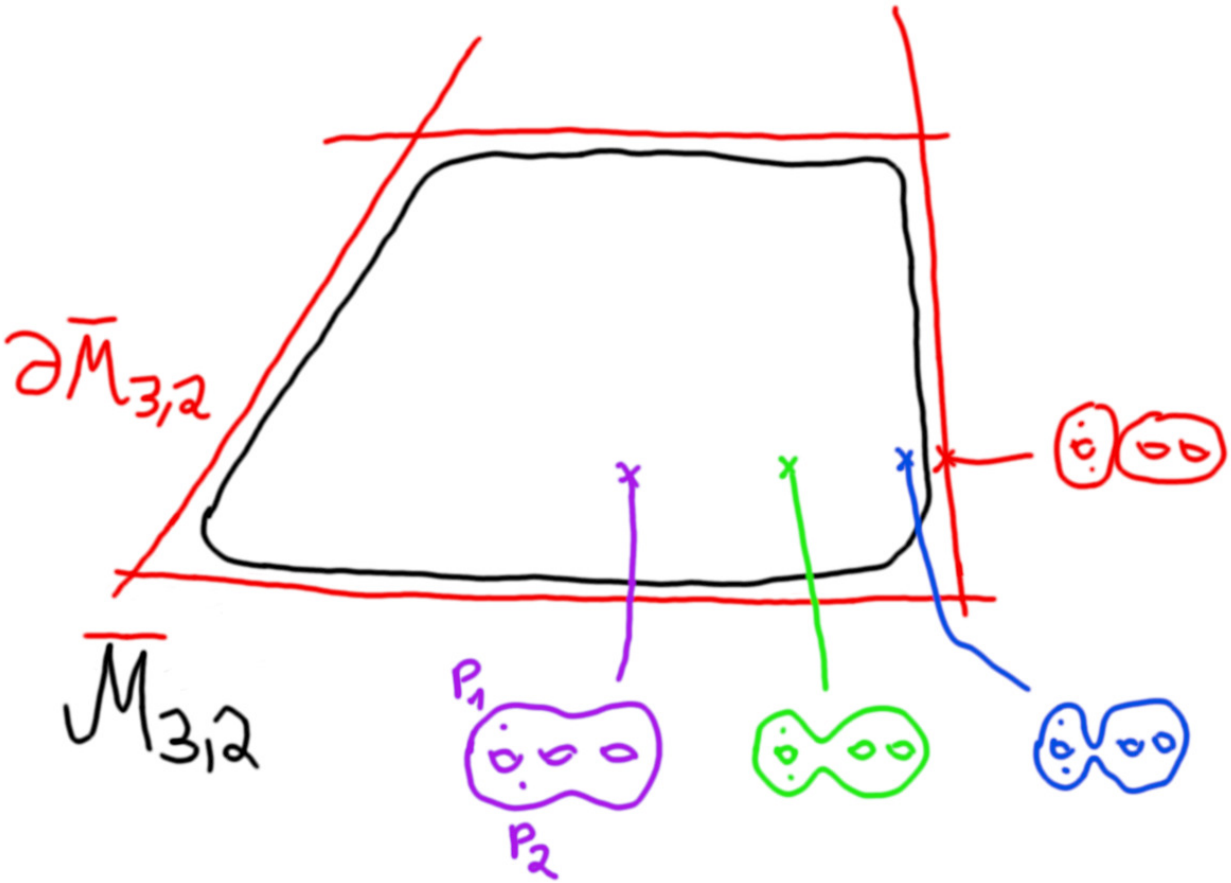

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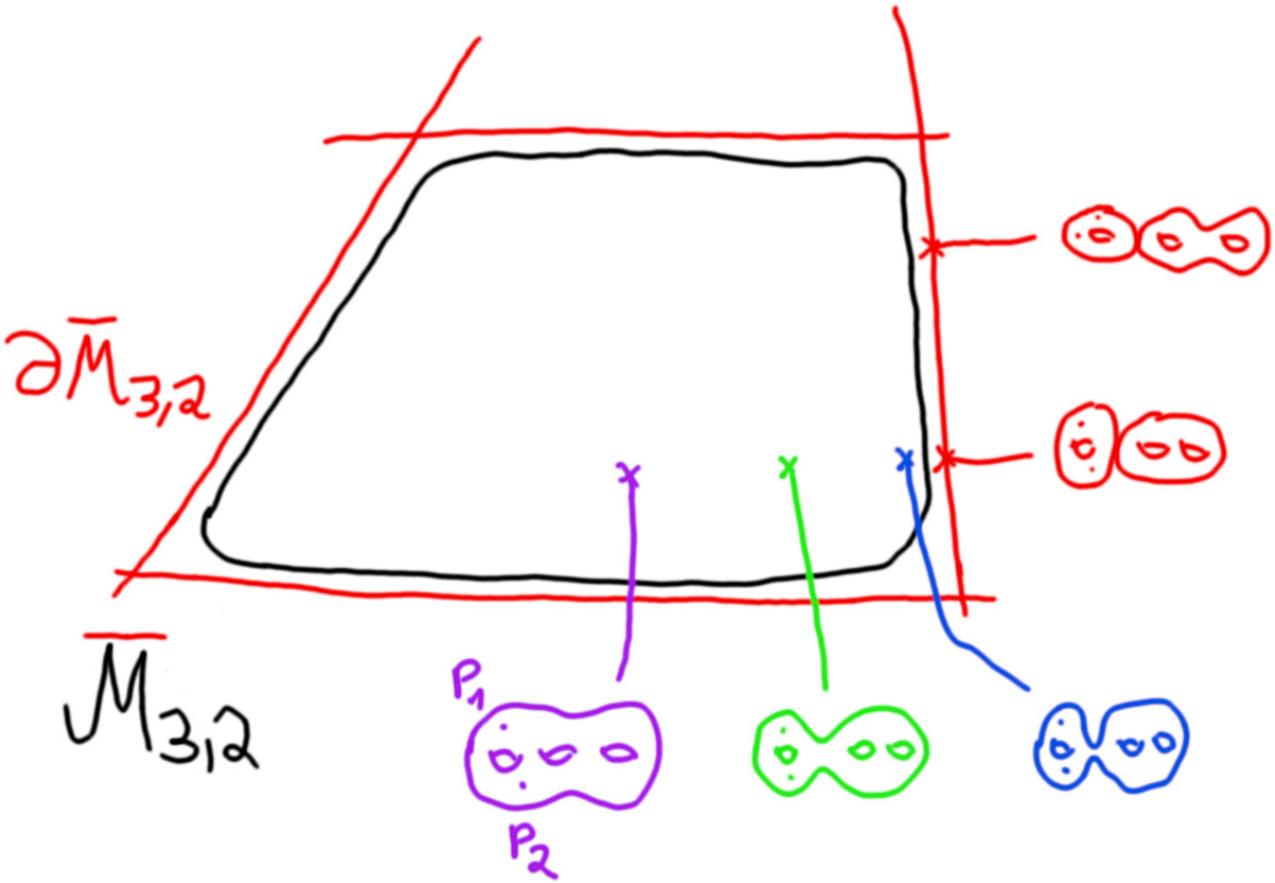

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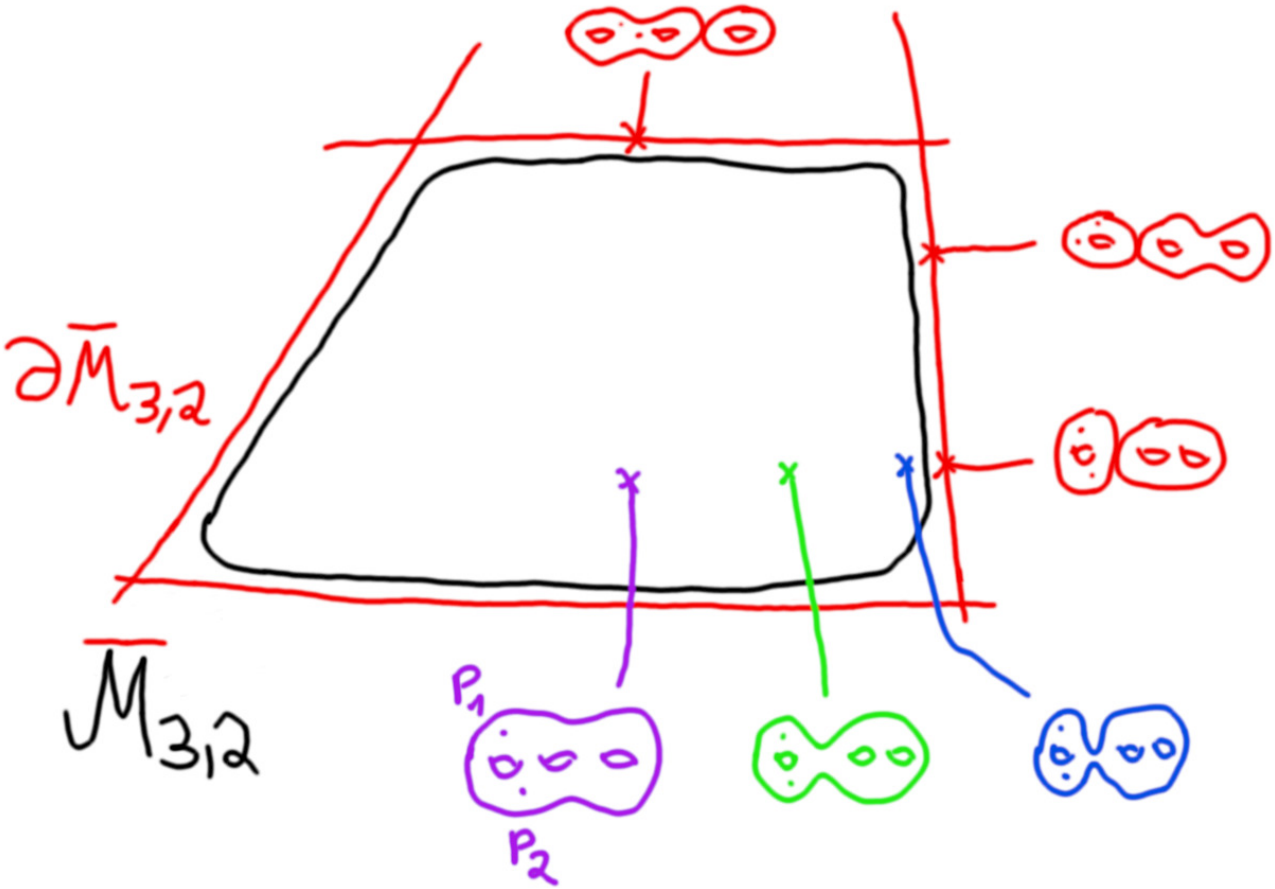

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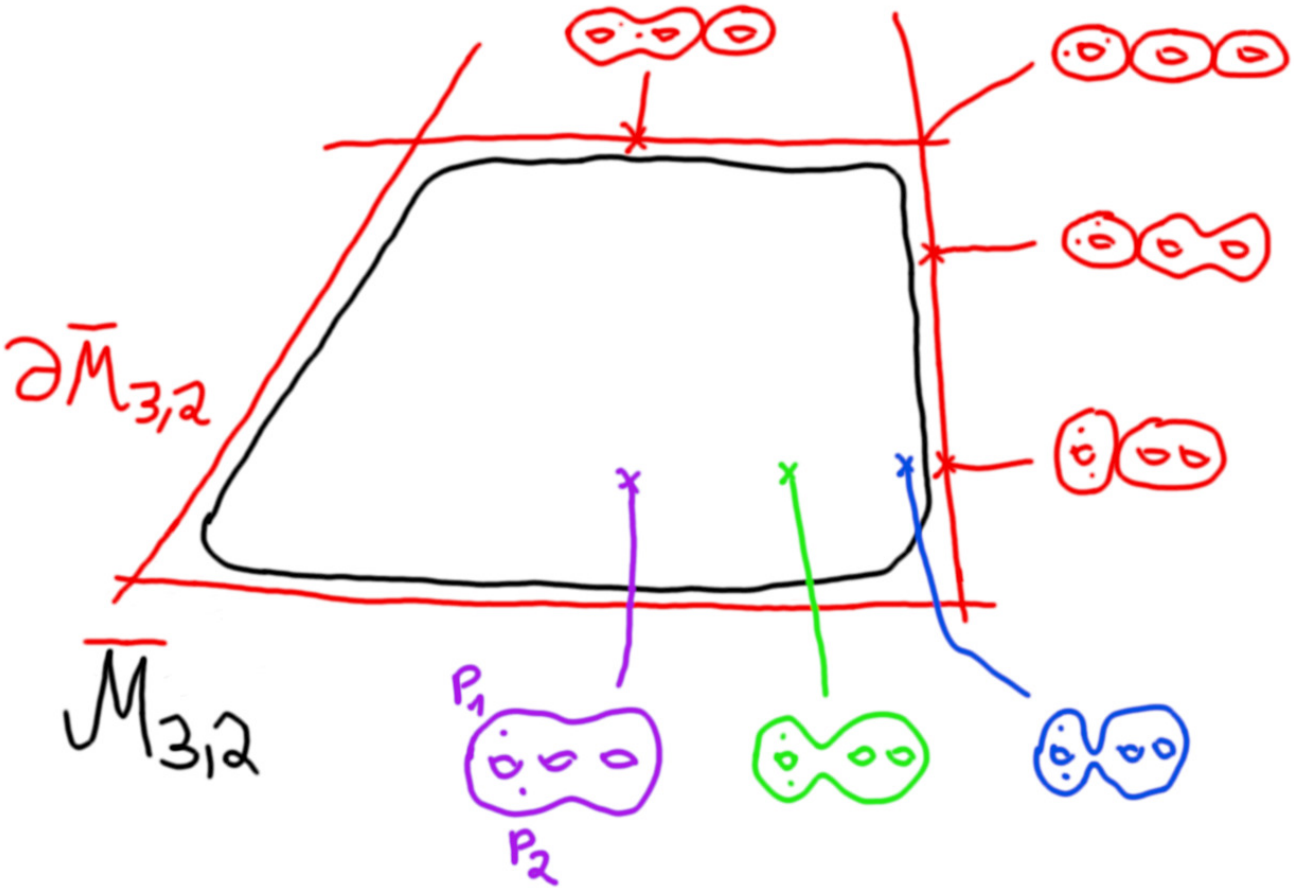

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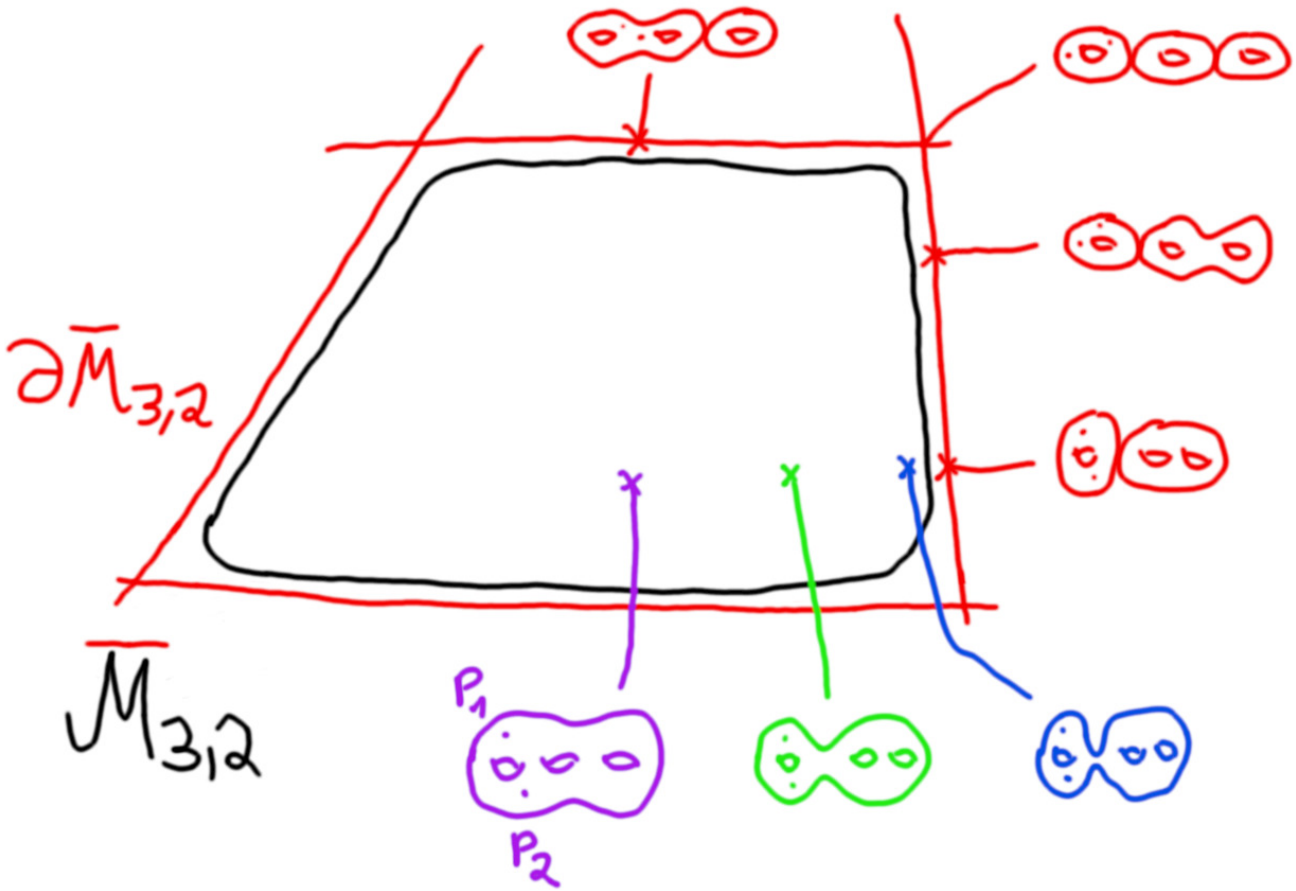
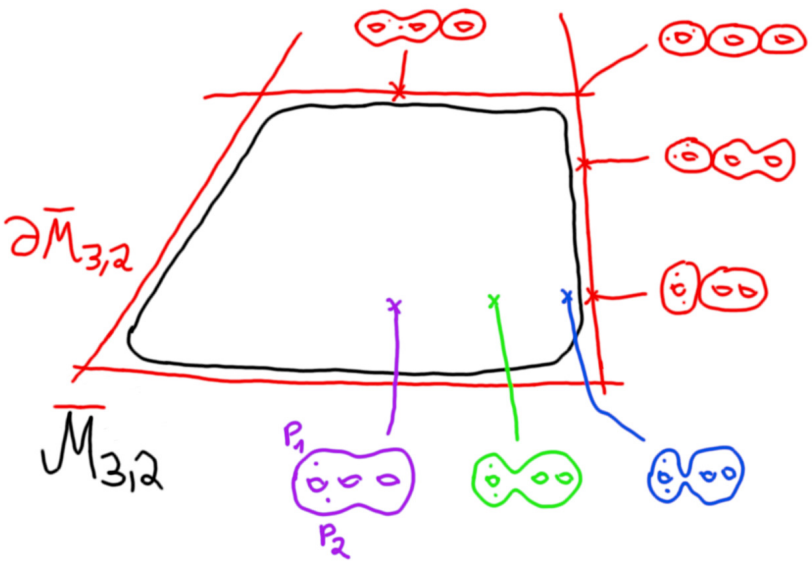


Fact

$\overline{\mathcal{M}}_{g,n}$ is smooth, compact, connected space of $\mathbb{C}$-dimension $3g-3+n$.

Tautological classes in the cohomology ring of $\overline{\mathcal{M}}_{g,n}$



Facts

Inside the cohomology $H^*(\overline{\mathcal{M}}_{g,n})$ there are natural *tautological classes* $[\Gamma, \alpha]$, indexed by certain decorated graphs.

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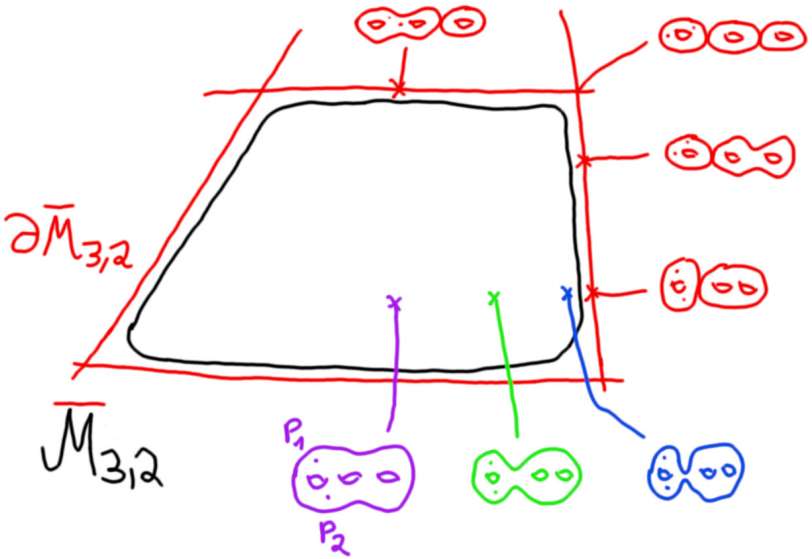

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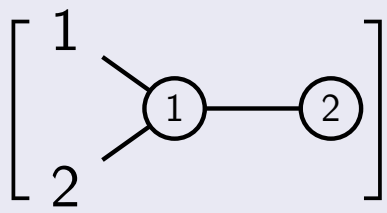

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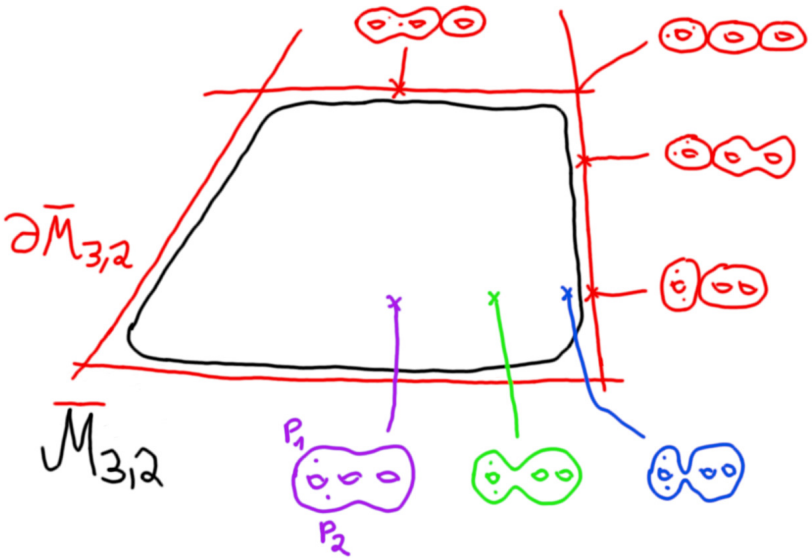

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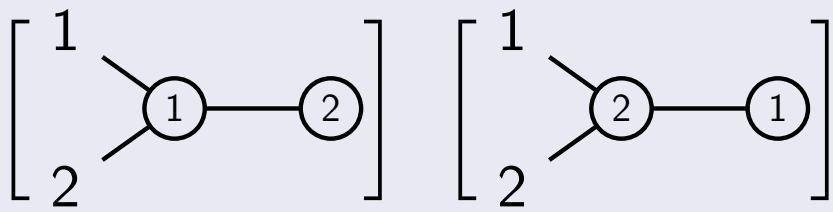

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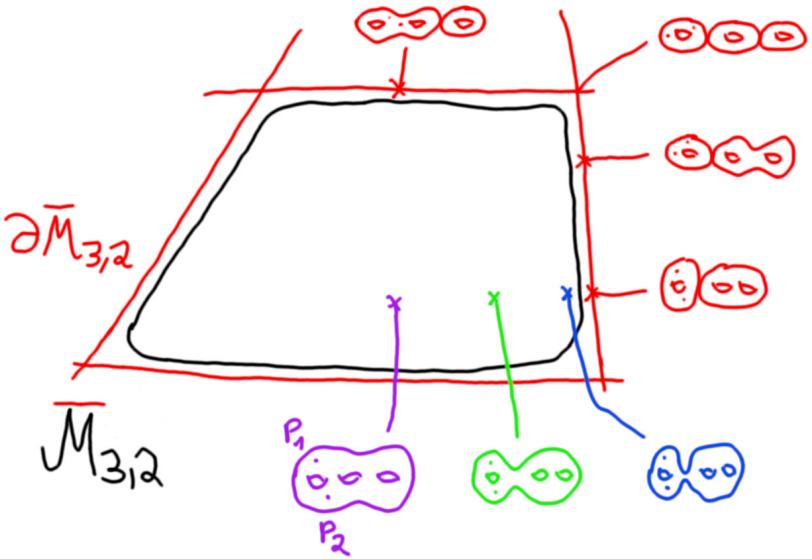

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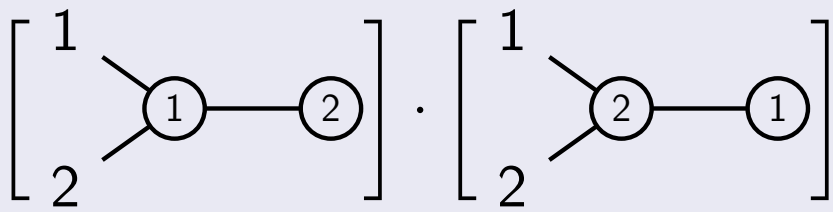

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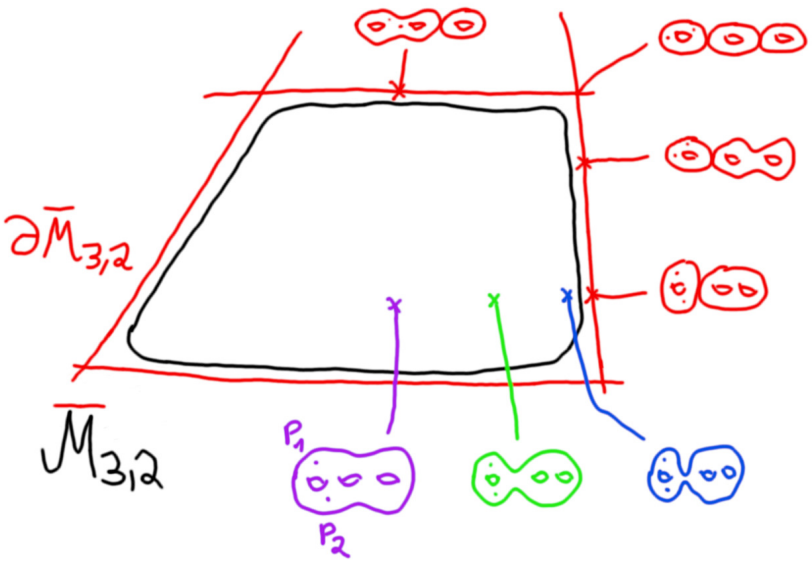

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Example

$$\left[\begin{smallmatrix}1\\2\end{smallmatrix}\right.\!\!\begin{array}{ccc} & 1 & \\ & & \end{array}\!\!\begin{array}{ccc} & & \\ & 2 & \end{array}\!\!\left.\begin{smallmatrix}\\\\\end{smallmatrix}\right] \cdot \left[\begin{smallmatrix}1\\2\end{smallmatrix}\right.\!\!\begin{array}{ccc} & 2 & \\ & & \end{array}\!\!\begin{array}{ccc} & & \\ & 1 & \end{array}\!\!\left.\begin{smallmatrix}\\\\\end{smallmatrix}\right]$$

$$= \left[\begin{smallmatrix}1\\2\end{smallmatrix}\right.\!\!\begin{array}{ccccc} & 1 & & 1 & \\ & & & & \end{array}\!\!\begin{array}{ccc} & & \\ & 1 & \end{array}\!\!\left.\begin{smallmatrix}\\\\\end{smallmatrix}\right]$$

The tautological ring $RH^*(\overline{\mathcal{M}}_{g,n}) \subset H^*(\overline{\mathcal{M}}_{g,n})$

Properties

- explicit, finite list of generators $[\Gamma, \alpha]$ as $\mathbb{Q}$-vector space
- combinatorial description of intersection product $[\Gamma, \alpha] \cdot [\Gamma', \alpha']$ (Graber-Pandharipande, 2003)
- list of many linear relations between the generators (Faber-Zagier 2000, Pandharipande-Pixton 2010, Pixton 2012, Pandharipande-Pixton-Zvonkine 2013)
- effective description of isomorphism $RH^{\dim}(\overline{\mathcal{M}}_{g,n}) \cong \mathbb{Q}$ (Witten 1991, Kontsevich 1992)

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The `admcycles`-package

All of these (and more!) are now implemented and available in the Sage-package `admcycles`.

Mathematical connections and applications

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Example: Strata of differentials, flat surfaces

- Strata of abelian, quadratic, ... differentials naturally live in projectivized Hodge bundle over $\overline{\mathcal{M}}_{g,n}$ [BCG$^+$18, BCG$^+$19]
- Recursive description of their cohomology classes in terms of tautological classes [Sau19]
- Description of Masur-Veech volumes for strata of abelian differentials in terms of intersection numbers [CMSZ19]

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Further examples

- Mapping class group
- Ribbon graphs
- Tropical geometry
- Integrable systems
- Gromov-Witten invariants and enumerative geometry

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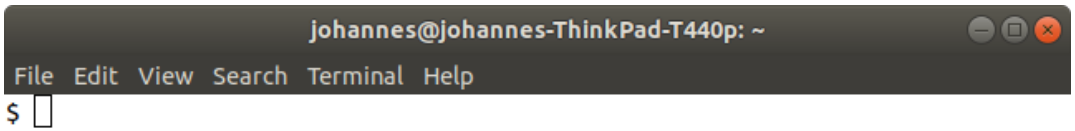
History and authors

- Based on earlier implementation by Aaron Pixton
- Interface and extension by Johannes Schmitt and Jason van Zelm for study of *admissible cover cycles* ([Sv18]) – hence the name
- Conversion to proper Sage-package and lots of improvements by Vincent Delecroix
- Available on `https://gitlab.com/jo314schmitt/admcycles`

A first computation



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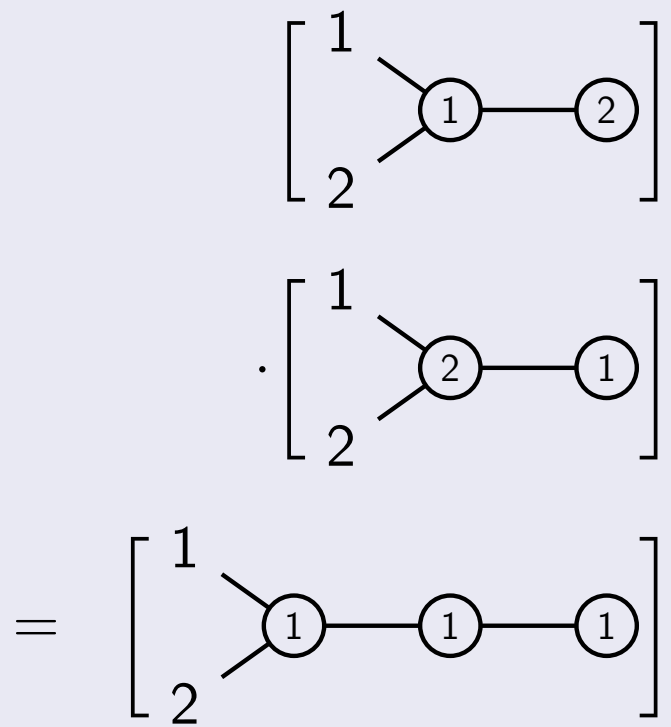

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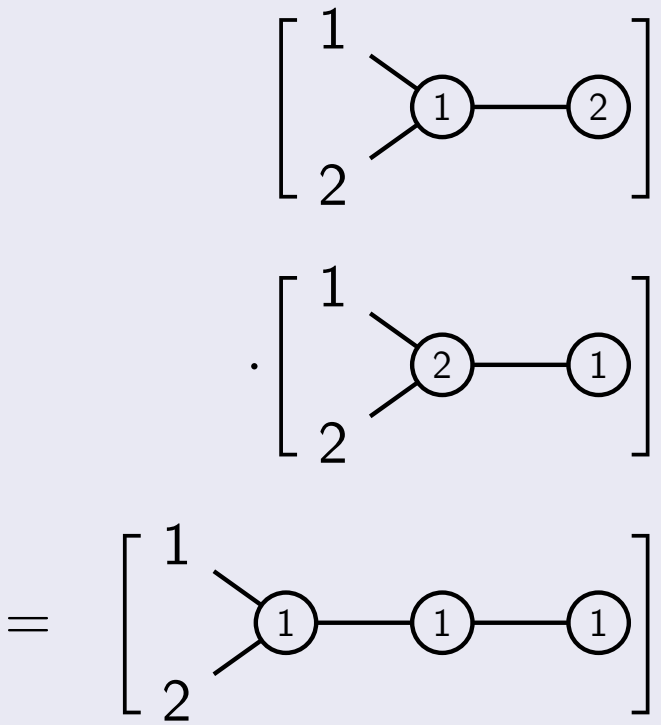

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johannes@johannes-ThinkPad-T440p: ~
File  Edit  View  Search  Terminal  Help
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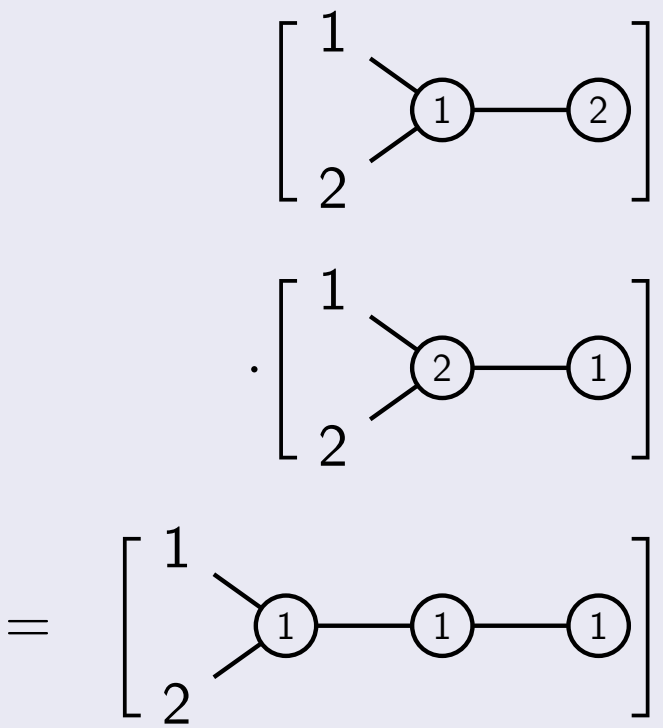

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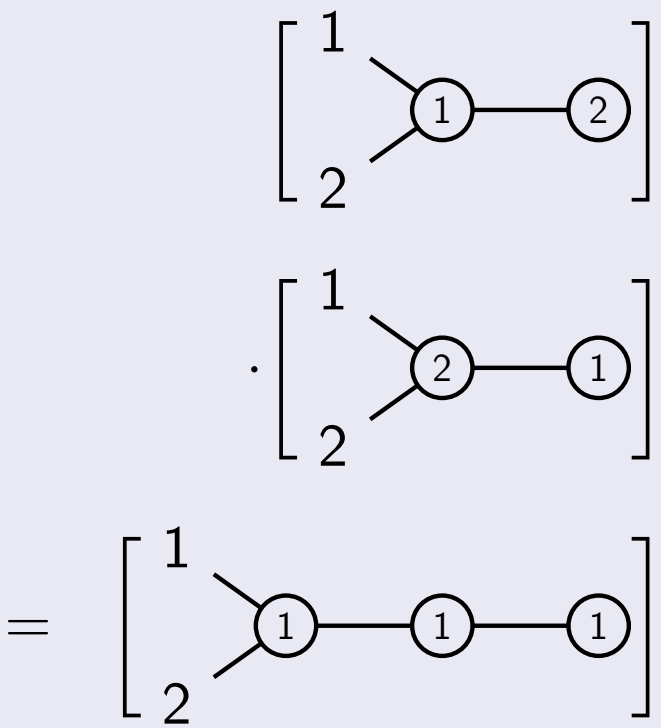

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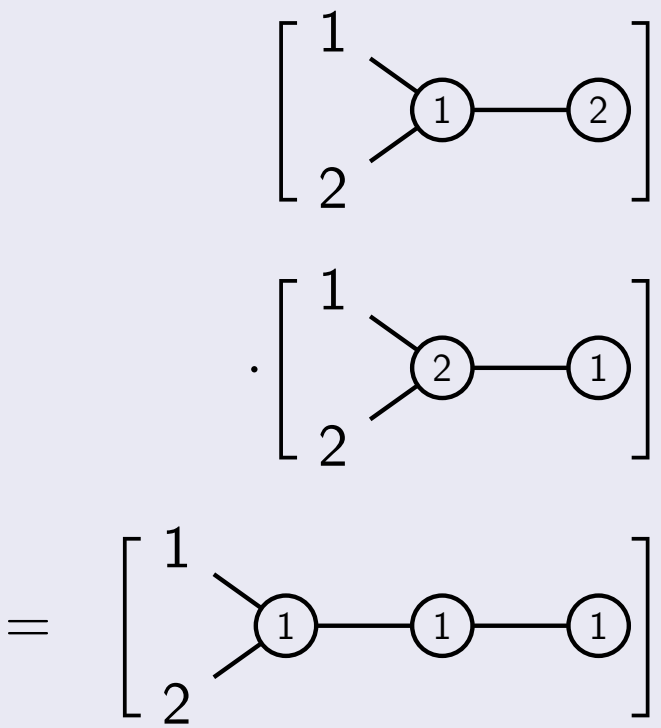

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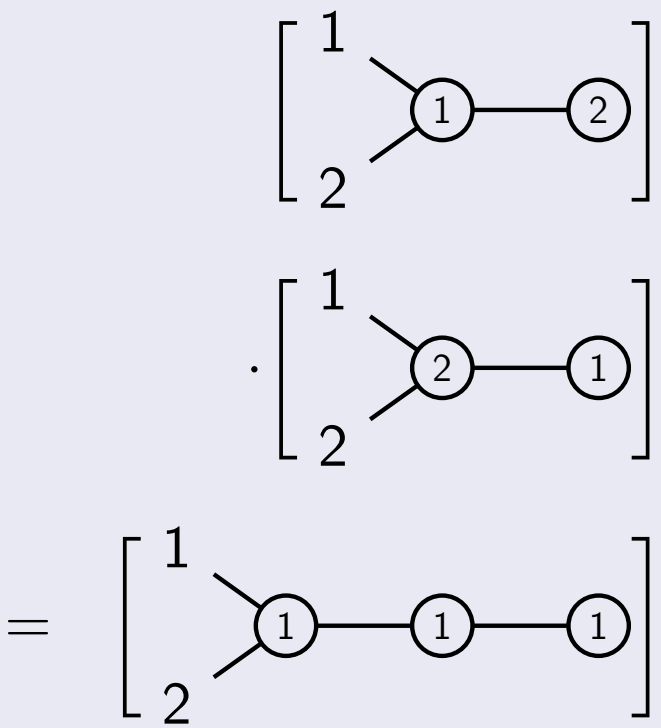

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sage: from admcycles import *
sage: reset_g_n(3,2)
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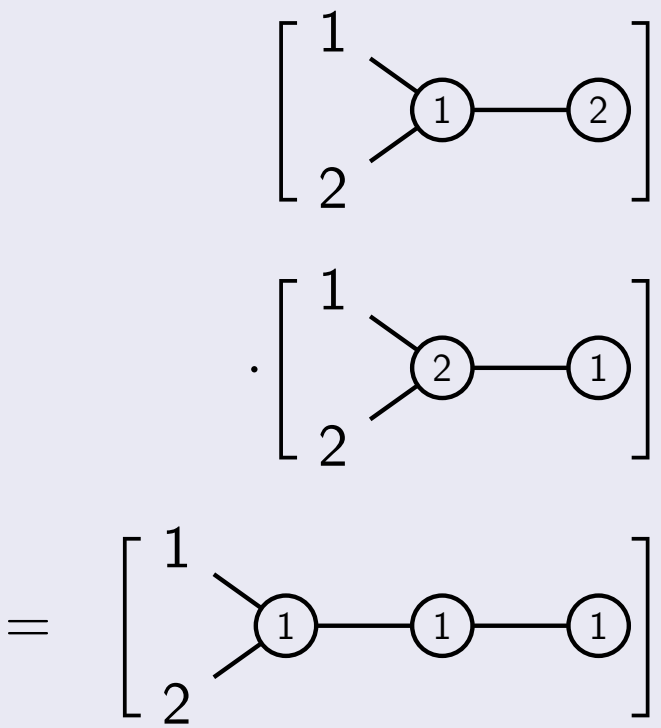

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sage: from admcycles import *
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sage: A=sepbdiv(1,(1,2)); A
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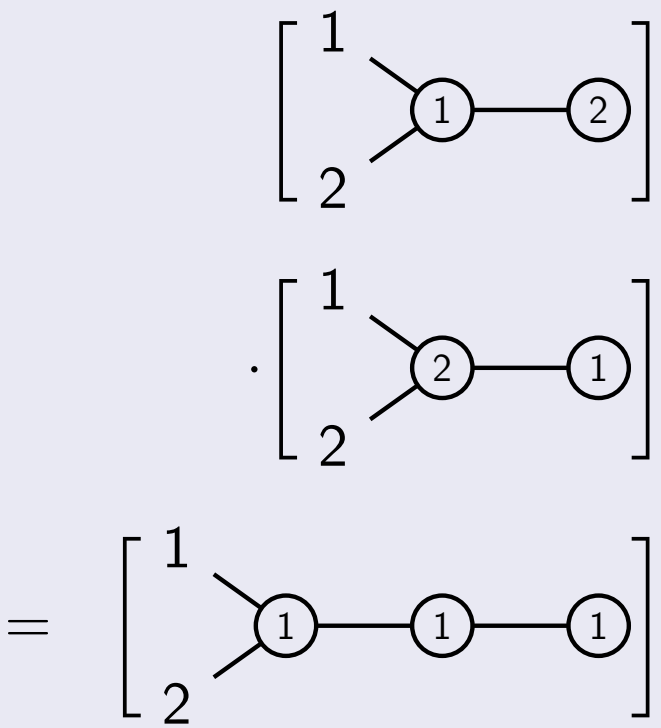

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sage: from admcycles import *
sage: reset_g_n(3,2)
sage: A=sepbdiv(1,(1,2)); A
Graph :      [1, 2] [[1, 2, 3], [4]] [(3, 4)]
Polynomial : 1*
sage:
```

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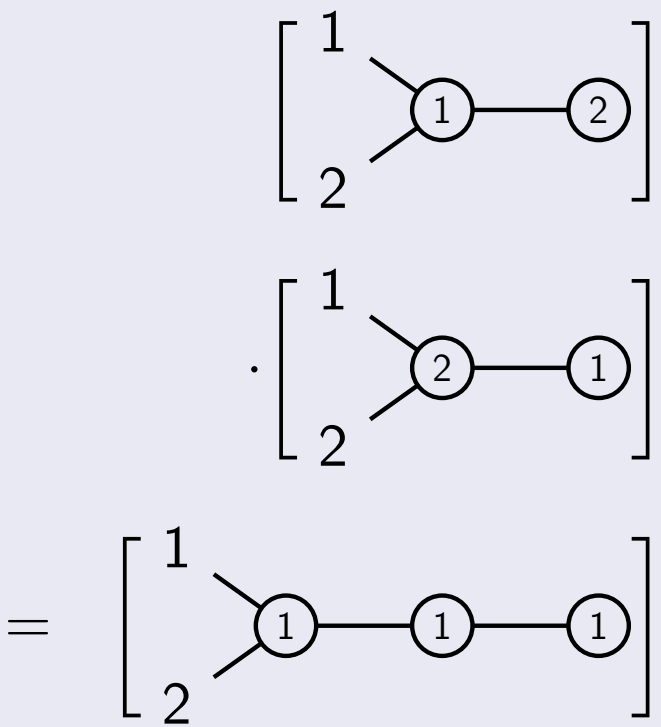

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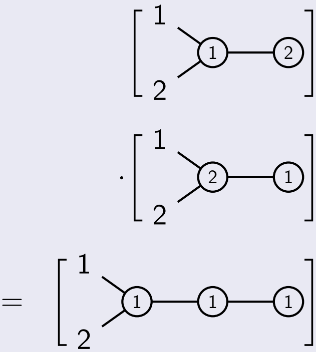

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sage: B=sepbdiv(2,(1,2)); B
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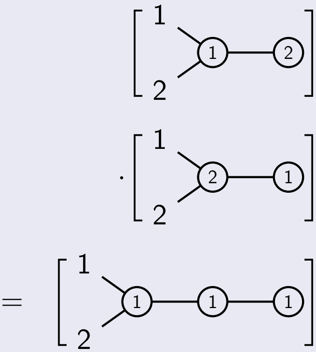
Polynomial : 1*
sage: A*B
```

Example



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sage: A*B
Graph :      [1, 1, 1] [[1, 2, 3], [5], [4, 6]] [(3, 4), (5, 6)]
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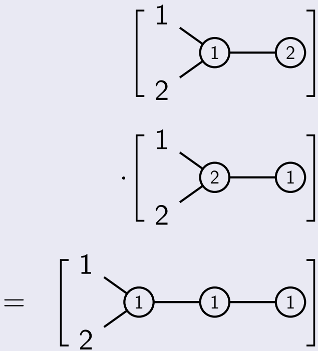
Graph :      [1, 0, 2] [[5], [1, 2, 3, 6], [4]] [(3, 4), (5, 6)]
Polynomial : 1*
sage: 
```

Example



A first computation

IPython: home/johannes

File Edit View Search Terminal Help

```
$ source /usr/share/sagemath/bin/sage-env
$ pip install git+https://gitlab.com/jo314schmitt/admcycles
Collecting git+https://gitlab.com/jo314schmitt/admcycles
  Cloning https://gitlab.com/jo314schmitt/admcycles to /tmp/pip-ZVvuax
-build
Installing collected packages: sage-sample
  Running setup.py install for sage-sample ... done
Successfully installed sage-sample-0.1
$ sage
```

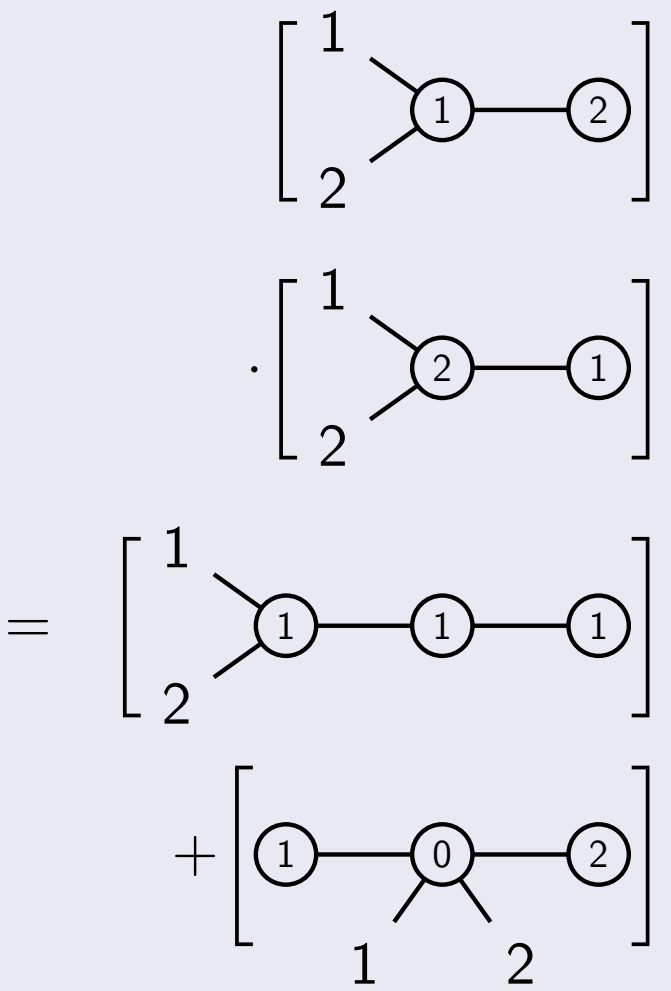
```
SageMath version 8.1, Release Date: 2017-12-07
Type "notebook()" for the browser-based notebook interface.
Type "help()" for help.
```

```
sage: from admcycles import *
sage: reset_g_n(3,2)
sage: A=sepbdiv(1,(1,2)); A
Graph :      [1, 2] [[1, 2, 3], [4]] [(3, 4)]
Polynomial : 1*
sage: B=sepbdiv(2,(1,2)); B
Graph :      [2, 1] [[1, 2, 3], [4]] [(3, 4)]
Polynomial : 1*
sage: A*B
Graph :      [1, 1, 1] [[1, 2, 3], [5], [4, 6]] [(3, 4), (5, 6)]
Polynomial : 1*

Graph :      [1, 0, 2] [[5], [1, 2, 3, 6], [4]] [(3, 4), (5, 6)]
Polynomial : 1*
sage:
```

I forgot a term!

Example



Useful resources and further reading

- This presentation (see also appendix)
- `https://gitlab.com/jo314schmitt/admcycles`
  - `README.rst` – detailed installation instructions
  - `examples.sage` – example computations with comments
- `https://people.math.ethz.ch/~schmittj/manual.pdf`
  User manual with explanations of basic functions and sample computations
- `http://www-personal.umich.edu/~janda/taut.pdf`
  Short mathematical introduction of the tautological ring

Applications of admcycles (and Pixton's program)

- Verifying theorems in special cases
  - New recursion formulas for intersection numbers [GHW19]
  - Bielliptic Hodge integrals [PT17]
- Falsifying theorems in special cases
- Computing formulas for interesting cycle classes
  - Admissible cover cycles [Sv18]
- Exploring conjectures
  - Recursion for fundamental class of strata of differentials [FP18, Sch18]
  - Intersections of strata of differentials [HPS17]
  - Recursion for hyperelliptic cycles (work in progress with Renzo Cavalieri)

Project proposals

Project proposals

Improving cache function

- Problem: currently, working memory (not time!) is a bottleneck for many computations due to handwritten cache function
- Skills: basic Sage programming; Time horizon: 3-7 hours
- Possible Impact: access to new computations (e.g. hyperelliptic cycle in $\overline{\mathcal{M}}_7$)

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Improving computation of Double Ramification Cycle

- Problem: currently compute set of flows (in $\mathbb{Z}/r\mathbb{Z}$) on graph by brute-force trial-and-error, limiting number of accessible cases
- Skills: graph algorithms; Time horizon: 1-3 days
- Possible Impact: active research field (Buryak-Rossi; Wu; Pandharipande, ...) with interest in doing concrete computations

Project proposals

Implement conjectural recursion for fundamental class of strata of differentials

- Problem: the papers [FP18, Sch18] propose an algorithm for recursively computing classes of strata of differentials, which has yet to be implemented
- Skills: basic familiarity with moduli of curves, operations in tautological ring as implemented in `admcycles`
- Time horizon: 1-3 days
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Thank you for your attention!

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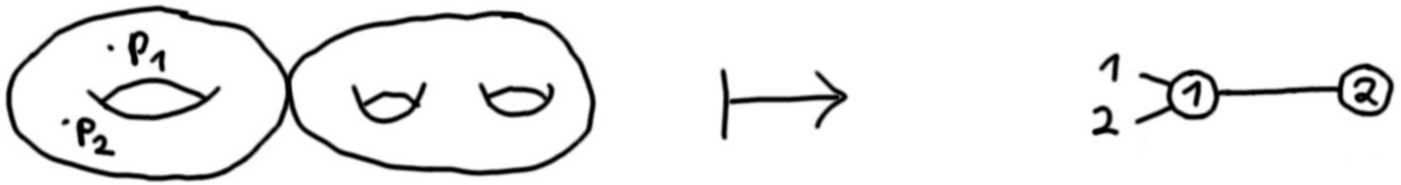
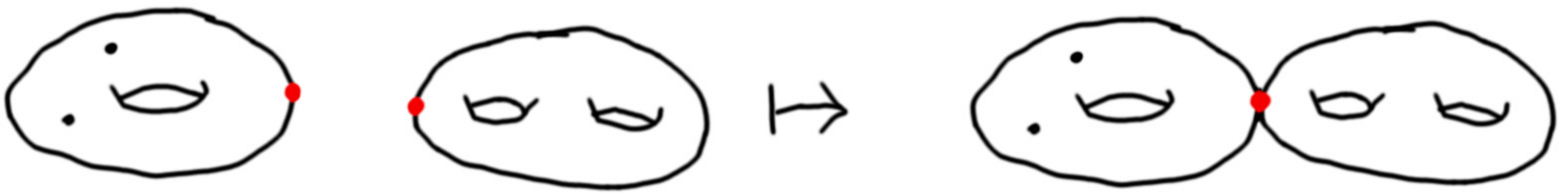
Recursive boundary structure

To $(C, p_1, \ldots, p_n) \in \overline{\mathcal{M}}_{g,n}$ we can associate a stable graph $\Gamma_{(C,p_1,\ldots,p_n)}$

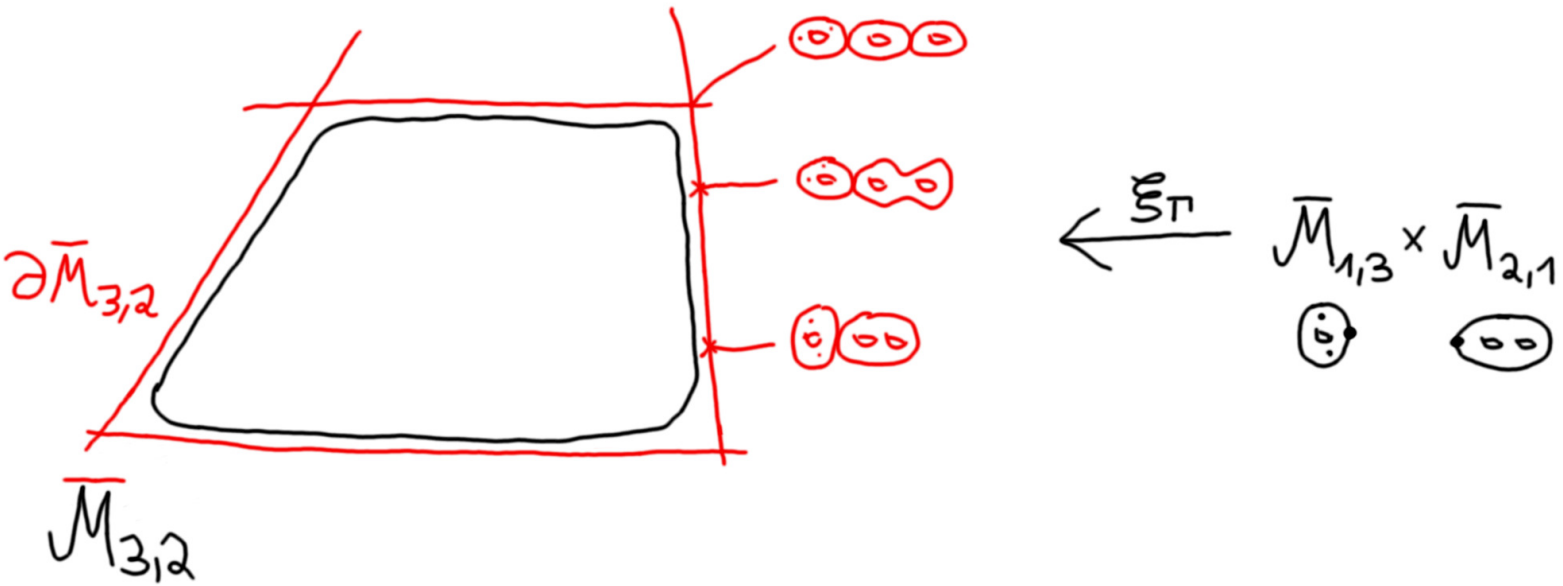


Conversely, given a stable graph Γ we have a gluing map

$$\xi_\Gamma : \prod_{v \in V(\Gamma)} \overline{\mathcal{M}}_{g(v),n(v)} = \overline{\mathcal{M}}_{1,3} \times \overline{\mathcal{M}}_{2,1} \to \overline{\mathcal{M}}_{3,2}$$



Recursive boundary structure



Proposition

The map ξ_Γ is finite with image equal to

$$\overline{\{(C, p_1, \ldots, p_n) : \Gamma_{(C,p_1,\ldots,p_n)} = \Gamma\}}.$$

The tautological ring

Definition: the tautological ring

The tautological ring $RH^*(\overline{\mathcal{M}}_{g,n}) \subset H^*(\overline{\mathcal{M}}_{g,n})$ is spanned as a $\mathbb{Q}$-vector subspace by elements

$$[\Gamma, \alpha] = (\xi_\Gamma)_* \left(\underbrace{\text{product of } \kappa, \psi\text{-classes}}_{\alpha} \text{ on } \prod_{v \in V(\Gamma)} \overline{\mathcal{M}}_{g(v),n(v)} \right)$$

Example

$$\left[\begin{array}{l} 1 \\ \\ 2 \end{array} \right. \overset{\kappa_1}{\underset{}{\textcircled{1} \longrightarrow \textcircled{2}}} \left. \right] = (\xi_\Gamma)_* (\kappa_1 \otimes \psi_h) \in RH^*(\overline{\mathcal{M}}_{3,2}),$$

for $\xi_\Gamma : \overline{\mathcal{M}}_{1,3} \times \overline{\mathcal{M}}_{2,1} \to \overline{\mathcal{M}}_{3,2}$ and $\alpha = \kappa_1 \otimes \psi_h \in H^*(\overline{\mathcal{M}}_{1,3} \times \overline{\mathcal{M}}_{2,1})$

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References I

- Matt Bainbridge, Dawei Chen, Quentin Gendron, Samuel Grushevsky, and Martin Möller, *Compactification of strata of Abelian differentials*, Duke Math. J. 167 (2018), no. 12, 2347–2416. MR 3848392
- ———, *Strata of k-differentials*, Algebr. Geom. 6 (2019), no. 2, 196–233. MR 3914751
- Dawei Chen, Martin Möller, Adrien Sauvaget, and Don Zagier, *Masur-Veech volumes and intersection theory on moduli spaces of abelian differentials*, arXiv e-prints (2019), arXiv:1901.01785.
- Gavril Farkas and Rahul Pandharipande, *The moduli space of twisted canonical divisors*, J. Inst. Math. Jussieu 17 (2018), no. 3, 615–672. MR 3789183
- Harald Grosse, Alexander Hock, and Raimar Wulkenhaar, *A Laplacian to compute intersection numbers on $\overline{\mathcal{M}}_{g,n}$ and correlation functions in NCQFT*, arXiv e-prints (2019), arXiv:1903.12526.

References II

- D. Holmes, A. Pixton, and J. Schmitt, *Multiplicativity of the double ramification cycle*, ArXiv e-prints (2017), accepted by Doc. Math.
- Rahul Pandharipande and Hsian-Hua Tseng, *Higher genus Gromov-Witten theory of the Hilbert scheme of points of the plane and CohFTs associated to local curves*, arXiv e-prints (2017), arXiv:1707.01406.
- Adrien Sauvaget, *Cohomology classes of strata of differentials*, Geom. Topol. 23 (2019), no. 3, 1085–1171. MR 3956890
- Johannes Schmitt, *Dimension theory of the moduli space of twisted k-differentials*, Doc. Math. 23 (2018), 871–894. MR 3861042
- Johannes Schmitt and Jason van Zelm, *Intersections of loci of admissible covers with tautological classes*, arXiv e-prints (2018), arXiv:1808.05817.